About the Cover:
The cover picture shows a three-dimensional perspective view of JASON, a remotely operated underwater vehicle in relation to its target of interest, the USS Scourge. The sunken ship, part of the US Great Lakes fleet during the war of 1812, was the site of a remote underwater archaeological survey during the spring of 1990. The photograph was produced by Marquest Group, Inc. (see the invited paper by W.K. Stewart in this volume).

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Preface

This volume presents the proceedings of the 9th International Conference of the Computer Graphics Society, CG International '91, Scientific Visualization of Physical Phenomena, held at the Massachusetts Institute of Technology in Cambridge, Massachusetts in the United States of America, on June 26-28, 1991. Since its foundation in 1983, this conference has continued to attract high quality research articles in all aspects of computer graphics and its applications. Previous conferences in this series were held in Japan (1983-1987), in Switzerland (1988), in the United Kingdom (1989), and in Singapore (1990). Future CG International conferences are planned in Japan (1992), Switzerland (1993), and in Australia (1994).

The title of this book, Scientific Visualization of Physical Phenomena, reflects the special emphasis of this year’s CG International Conference. Visualization in science and engineering is rapidly developing into a vital area because of its potential for significantly contributing to the understanding of physical processes and the design automation of man-made systems. With the increasing emphasis on handling complexity in physical and artificial processes and systems, and with continuing advances in specialized graphics hardware and in processing software and algorithms, visualization is expected to play an increasingly dominant role in the foreseeable future.

The keynote paper, the five invited papers, and the 31 refereed papers included in this book represent the state-of-the-art in scientific visualization. These papers are grouped into 11 chapters. The keynote and invited papers in the first chapter address leading-edge research aspects of visualization from the perspectives of computer animation; simulation and modeling, volume visualization; visualization in astrophysics; computational geometry; engineering design and analysis; and ocean sciences and engineering. The refereed papers were selected, after peer review, from a large number of papers submitted from around the world and are included in Chapters 2 to 11, entitled as follows: Animation; Parallel Processing; Volume Rendering; Visualization Methods; Ray Tracing/Rendering; Picture Generation; Computational Geometry; Visualization in Engineering, Fluid Flow Visualization; and Applications. Countries represented in this volume include Australia, Belgium, Canada, Germany, Italy, Japan, New Zealand, Singapore, Switzerland, the United Kingdom, and the United States of America. Thus, there is wide international coverage of research in this important emerging area.

This volume concludes with a listing of the CG International '91 conference committees, our staff, cooperating societies, sponsors, and the technical reviewers. The efforts, support, and contributions of all these individuals and organizations are gratefully acknowledged. Special thanks are due to Drs. Chryssostomos Chryssostomidis, Rae A. Earnshaw, Bertram Herzog, and
Tosiyasu L. Kunii for their support, advice, and organizational assistance and to Ms. Marge Chryssostomidis for editorial assistance in compiling these proceedings. Finally, special appreciation is due to Ms. Barbara Dullea, director of the CG International '91 Secretariat, who expertly assisted us in organizing CG International '91 and in making it a resounding success.

Nicholas M. Patrikalakis
Table of Contents

Chapter 1: Keynote and Invited Papers

Visualization: New Concepts and Techniques to Integrate Diverse Application Areas
T.L. Kunii and Y. Shinagawa .......................................................... 3

Introduction to Volume Synthesis
A. Kaufman .................................................................................. 25

Computer Visualization in Spacecraft Exploration of the Solar System
W.R. Thompson and C. Sagan ......................................................... 37

Computational Geometry and Visualization: Problems at the Interface
L.J. Guibas ..................................................................................... 45

Visualization for Engineering Design
H. Nowacki .................................................................................... 61

Visualization Resources and Strategies for Remote Subsea Exploration
W.K. Stewart .................................................................................. 85

Chapter 2: Animation

A Particle-Based Computational Model of Cloth Draping Behavior
D.E. Breen, D.H. House, and P.H. Getto ....................................... 113

Physically-Based Interactive Camera Motion Control Using 3D Input Devices
R. Turner, F. Balaguer, E. Gobbetti, and D. Thalmann ............ 135

Aspects of Motion Design for Physically-Based Animation
D. Haumann, J. Weichert, K. Arya, B. Bacon, A. Khorasani, A. Norton, and P. Sweeney ..................... 147
Chapter 3: Parallel Processing

Terrain Perspectives on a Massively Parallel SIMD Computer
G. Vézina and P.K. Robertson ................................................................. 163

Surface Tree Caching for Rendering Patches in a Parallel Ray Tracing System
W. Lamotte, K. Elen, and E. Flerackers ........................................ 189

Chapter 4: Volume Rendering

Context Sensitive Normal Estimation for Volume Imaging
R. Yagel, D. Cohen, and A. Kaufman ................................................. 211

Rapid Volume Rendering Using a Boundary-Fill Guided Ray Cast Algorithm
P.M. Hall and A.H. Watt ................................................................. 235

A 3D Surface Construction Algorithm for Volume Data
R. Shu and R.C. Krueger ................................................................. 251

Chapter 5: Visualization Methods

Compositional Analysis and Synthesis of Scientific Data Visualization Techniques
H. Senay and E. Ignatius ................................................................. 269

Precise Rendering Method for Edge Highlighting
T. Tanaka and T. Takahashi ........................................................... 283

Fractals Based on Regular Polygons and Polyhedra
H. Jones and A. Campa ................................................................. 299

Chapter 6: Ray Tracing/Rendering

Ray Tracing Gradient Index Lenses
K.G. Suffer and P.H. Getto .............................................................. 317

Shapes and Textures for Rendering Coral
N.L. Max and G. Wyvill ................................................................. 333

A New Color Conversion Method for Realistic Light Simulation
T. Naka, K. Nishimura, F. Taguchi, and Y. Nakase ......................... 345
Chapter 7: Picture Generation

Two-Dimensional Vector Field Visualization by Halftoning
R.V. KLASSEN and S.J. HARRINGTON ................................................. 363

Three Plus Five Makes Eight: A Simplified Approach to Halftoning
G. WYVILL and C. MCNAUGHTON ...................................................... 379

Chapter 8: Computational Geometry

A Theory of Geometric Contact for Computer Aided Geometric Design of Parametric Curves
C. LEE, B. RAVANI, and A.T. YANG ................................................. 395

Generalization of a Family of Gregory Surfaces
K. UEDA and T. HARADA ................................................................... 417

A New Control Method for Free-Form Surfaces with Tangent Continuity and its Applications
K. KONNO, T. TAKAMURA, and H. CHIYOKURA ................................. 435

Hybrid Models and Conversion Algorithms for Solid Object Representation
L. DE FLORIANI and E. PUPPO ............................................................ 457

Surface Generation Using Implicit Cubics
B. GUO ................................................................................................. 485

Chapter 9: Visualization in Engineering

Equilibrium and Interpolation Solutions Using Wavelet Bases
A.P. PENTLAND .................................................................................... 507

Dynamic 3D Illustrations with Visibility Constraints
S.K. FEINER and D.D. SELIGMANN .................................................... 525

Piecewise Linear Approximations of Digitized Space Curves with Applications
I. IHM and B. NAYLOR ....................................................................... 545

Chapter 10: Fluid Flow Visualization

Ellipsoidal Quantification of Evolving Phenomena
D. SILVER, N. ZABUSKY, V. FERNANDEZ, M. GAO, and R. SAMTANEY ................................................................. 573
Smoothed Particle Rendering for Fluid Visualization in Astrophysics
M. NAGASAWA and K. KUWAHARA .................................................... 589

Chapter 11: Applications

Pan-Focused Stereoscopic Display Using a Series of Optical Microscope Images
K. KANEDA, S. ISHIDA, and E. NAKAMAE ......................................... 609

Reconstructing and Visualizing Models of Neuronal Dendrites
I. CARLBOM, D. TERZOPOULOS, and K.M. HARRIS ............................ 623

A Visualization and Simulation System for Environmental Purposes
M. GROSS and V. KÜHN ..................................................................... 639

Synchronized Acquisition of Three-Dimensional Range and Color Data and its Applications
Y. WATANABE and Y. SUENAGA ......................................................... 655

Piecewise Planar Surface Models from Sampled Data
D.A. SOUTHARD ................................................................................... 667

Conference Organization Committee ................................................ 681
List of Sponsors ................................................................................... 683
List of Technical Reviewers .................................................................. 685
List of Contributors ............................................................................. 687
Keyword Index .................................................................................... 689
Chapter 1

Keynote and Invited Papers
Visualization: New Concepts and Techniques to Integrate Diverse Application Areas

Tosiyasu L. Kunii and Yoshihisa Shinagawa

ABSTRACT

Visualization models are diverse as a natural reflection of the diversity of the highly growing application areas of visualization. This is a piece of work trying to integrate them into a small set of handy models based on a few crisp concepts. As one of such concepts, we take the Reeb graph and show how to build an integrated visualization model named ModelVisual serving to integrate the diversity. ModelVisual is an abstraction hierarchy of incrementally modular data structure which includes the topological, geometrical and other layers and the operators working on the layers to recognize, view and display them. The self-visualizing visualization model is provided for implementing ModelVisual. The application models integrated are: the homotopy model which again integrates the triangulation model, the spline surface model and the loft surface model; the singularity model applied to garment wrinkling; the tree model applied to forest growth. The implication of the integrated visualization model to the visual computer architecture through information localization is also clarified.

Keyword: integrated visualization model, Reeb graph, self-visualizing visualization model, homotopy, singularity, forest growth model, information locality, visual computer architecture

1. INTRODUCTION

This research is based on a belief after scientific tradition that the diversity in the appearance of phenomena and objects does not necessarily mean the diversity in the rules governing them. The rules sometimes are called models or theories. Usually a few basic rules do well as the quantum theory in physics and the periodic table in chemistry. The term "visualization" is used very popularly to signify the computer display of the appearances which are indeed diverse. Then, what are a few basic rules governing the diversity of visualization?

2. ModelVisual: AN INTEGRATED VISUALIZATION MODEL

As in the other established disciplines, the structure common to the appearances of the diverse phenomena and objects visualized provides the basis of the rules. It is a type of abstract data structure, hierarchically organized for modularity.

An incrementally modular visual structure can be built by having topology at the most abstract layer of the hierarchy, geometry at the next layer by adding the coordinate system and the measure to the top layer, and non structural information such as colors and mass to the bottom layer. The operators defined are: the layer specific operators which consists of the topological, geometrical and the other attribute operators, and the global operators which include the view, recognition, display and database operators.

The data structure thus defined serves actually as the basic visualization model and is named ModelVisual. For the topological layer, to go beyond what has been already proposed and developed while keeping the layer modular in itself, surface topology which can go beyond the graph
theoretical and combinatorial topology is incorporated. To be more specific, in addition to the Euler-Poincaré characteristic (see for example, do Carmo 1976) of a surface to model the visible surface and its changes in terms of the number of the vertices, edges, faces, holes and rings of the surface appearing in three dimensional modelers (see for example, Mäntylä and Sulonen 1982; Chiyokura and Kimura 1983; Chiyokura 1988), the mountaineer's equation (see for example, Griffiths 1981) as an aspect of the Morse theory (see for example, Milnor 1969) is utilized to model the surface curve characteristics and changes in terms of the number of the peaks, pits and passes of the surface, at the nodes of the Reeb graph (for details, see Thom 1988; Shinagawa, Kunii, Nomura, Okuno and Young 1990) which is playing the role of the generalized topological "skeleton" of the three dimensional surface structure of the object or phenomenon being visualized.

ModelVisual as a candidate basic visualization model is tested later against the cases of visualization applications, particularly the visualization of complex physical objects and phenomena such as human auditory ossicles, garment wrinkling and forest growth, and it is shown that ModelVisual does model the fundamental visual structure of all the cases and thus serves as the basic model.

At the end of the paper, the conceptual architecture of the visual computers is presented and discussed in the light of the information locality made possible by ModelVisual.

3. A NOTION OF SELF-VISUALIZING VISUALIZATION MODEL: ModelVisual AS A SELF-VISUALIZER

Let us now organize the basic visualization model ModelVisual introduced previously, and briefly sketch a small core concept for making it self-visualizing. Not an easy job, but necessary and worth considering it as "should be" one of the major targets of visualization. The basic computational methods of generating itself are known in different areas, including the early work of Von Neumann on the theory of self-reproducing automata (Von Neumann 1966) and the rather popular but partial realization of compiler-compiler tools, yacc and lex (see for example, Schreiner and Friedman, Jr. 1985; Pyster 1988). To the best of our knowledge, however, a self-visualizing visualization model is not known yet.

Let us first confirm that to visualize is for human being. Then, the notion of the self-visualizing visualization model means that the model contains the display information on all the structures, operators and their relationships so that the model displays itself for human being to recognize and select, out of what displayed, the operators and also the structure as the operand of the operators. The human being can, at least partially but hardly fully, delegate the recognition and/or selection operations to the model. The model always prompts, on the display screen, the course and results of the operations for further human interaction.

For realizing the evolutionary architecture of the model, ModelVisual is designed to make it incrementally modular, namely any evolution in the structure can be added on without affecting the existing structure. The layered hierarchical structure is adopted to implement the architecture. The model contains the followings: the structure layers I, the operators II, and the self-visualization mechanism III which is the implementation of the self-visualizing visualization model. The operators are grouped into two, intra layer operators II.A and inter layer operators II.B, and perform 8 categories of functions: define, transform, update, delete, search, recognize, select and display. The self-visualization mechanism III consists of the self-visualization administrator III.1, the two interfaces – the human interface III.2 and the model interface to interact with the rest of the model III.3 –, the self-visualizing symbol depository III.4 and the self-visualizing operators III.S, based on our early work on a menu generator (Kunii and Shirota 1989). A provision is also made for supporting visualization data sharing, prototyping and history management through a visualization database management system (Krishnan and Kunii 1990).

The real power of ModelVisual to integrate diverse application areas in visualization is in the abstraction hierarchy of the structure layer. As shown in the rest of the paper, the incremental modularity of the abstraction hierarchy allows all the information to be attached freely to the core information of the model which is in the topology layer. Actually, to represent the topological information of objects, the Reeb graph is used in our approach and other information is attached to the Reeb graph (Thom 1988; Shinagawa, Kunii, Nomura, Okuno and Young 1990). Before finishing the section, we list the structure layers of ModelVisual:
I. Structure layers

1.1. The topology layer
   Topological structures which are continuous as in the case of a topological space, discrete as in the case of a graph in graph theory, or both.

1.2. The geometry layer
   Geometrical structures which also are continuous as in the case of analytical surfaces, discrete as in the case of volume rendered images and tesselated textures, or both.

1.3. The non visual structure layer
   The other attributes of the structures, which are visible as colors, invisible as mass, or both.

4. REPRESENTATION OF TOPOLOGY BY A REEB GRAPH

We define the Reeb graph which ModelVisual uses to represent the topological "skeleton". George Reeb first introduced this graph in his thesis (for details, see Thom 1988; Shinagawa, Kunii, Nomura, Okuno and Young 1990). The Reeb graph is defined on a manifold. A manifold can be regarded as a space that has the same local properties as the Euclidean space and generalized to make two sets of parameters (coordinates) continuously related if there is the overlap of the two ranges of homeomorphic maps from planes to open pieces of a parametrically represented surface. The formal definition can be found, for example, in Armstrong 1983. For instance, a plane or a sphere is a 2D (two dimensional) manifold. In what follows, the surface of a 3D object is considered as a 2D manifold.

The definition of the Reeb graph is as follows:

DEFINITION

Let \( f: S \rightarrow \mathbb{R} \) be a real valued function on a manifold \( M \). The Reeb graph of \( f \) is the quotient space of the graph of \( f \) in \( M \times \mathbb{R} \) by the equivalence relation \( \sim \) given below:

\[
(X_1, f(X_1)) \sim (X_2, f(X_2))
\]

holds if and only if \( f(X_1) = f(X_2) \) and \( X_1, X_2 \) are in the same connected component of \( f^{-1}(f(X_1)) \). The equivalent class of \( X \) is denoted by \([X]\) in what follows. For convenience, we define a function \( \pi: M \rightarrow M \times \mathbb{R} / \sim \) as the function as

\[
\pi(X_1) = [X_1].
\]

That is, the two points on the graph \( (X_1, f(X_1)) \) and \( (X_2, f(X_2)) \) are represented as the same node \( \pi(f(X_1)) \) in the Reeb graph if the values of \( f \) are the same and they belong to the same connected component of the inverse image of \( f(X_1) \) (or \( f(X_2) \)). All points that belong to the same equivalent class of the original space is represented as a node in the quotient space such as the Reeb graph.

In this paper, the Reeb graph of the height function \( h(X) \) on the surface (the 2D manifold) of an object is considered. Here, \( h(X) \) gives the height of the point on the manifold \( X = (x_1, x_2, x_3) \)

where \( x_1, x_2, x_3 \in \mathbb{R} \), namely,

\[
h(x_1, x_2, x_3) = x_3.
\]

To be simple, we consider the height \( z \) of each point \( X = (x, y, z) \) on the surface. The Reeb graph of the height function on a surface is the quotient space of the graph \( (x, y, z) \) which identifies \( (x_1, y_1, z) \) and \( (x_2, y_2, z) \) if these two points are in the same connected component on the cross section of the surface at the height \( z \). For example, the Reeb graph of the height function on a torus shown in Fig. 1a is as in Fig. 1c. This is easy to see when we consider cross sectional planes perpendicular to \( z \) axis as in Fig. 1b; all the contour lines on each plane are represented as a node in the Reeb graph.

4.1 Mountaineer's Equation

The most important nodes of the Reeb graph are the nodes that represent the pits, passes and peaks.
Fig. 1. A torus and the critical points (a), the cross sections (b) and the Reeb graph (c)

Fig. 2. The cylindrical coordinate system on a garment
These points are called the critical points. If the numbers of pits, passes and peaks are denoted as #(pits), #(passes) and #(peaks) respectively, then

\[
#(\text{pits}) - #(\text{passes}) + #(\text{peaks}) = \chi(S).
\]

This equation is known as "the mountaineer's equation" and can be found in Griffiths (1976). \(\chi(M)\) is the Euler-Poincaré characteristic of the surface \(S\) which is related to the number \(g\) of holes (handles) through \(S\) as

\[
g = \frac{2 - \chi(S)}{2}.
\]

\(g\) is also called the genus of the surface \(S\). A torus, for example, has one peak, two passes and one pit as shown in Fig. 1. The Euler-Poincaré characteristic is 0 because its genus is 1. It is easy to see that the mountaineer's equation holds for a torus because

\[
\chi(\text{torus}) = 1 - 2 + 1 = 0.
\]

The discussions of the Euler-Poincaré characteristic can be found, for example, in do Carmo (1976). In the Reeb graph, \(g\) is the number of the loops of the graph.

The mountaineer's equation serves to maintain topological integrity when the shape of an object is changed by topological or geometrical shape operators. For example, when a pit is created on a surface, the mountaineer's equation makes sure that a pass is generated with it so that the genus of the surface remains unchanged.

5. INTEGRATED MODELING OF GEOMETRY BY ModelVisual

Having the Reeb graph in the top topological layer of the integrated visualization model ModelVisual, we now show how versatile the model becomes in uniformly modeling diverse visualization cases. To make the explanation clear, we look into one level lower layer of abstraction in the abstraction hierarchy of the model. Now the master of the scene is the Reeb graph, the scene is the second geometrical layer of the model. What we are actually showing is how different types of geometrical information can be modularly and incrementally associated with the Reeb graph in the first layer to form the second layer. With the Reeb graph it is simple. It is done by associating any geometrical information with the nodes of the Reeb graph.

Below, we list the diverse cases of geometrical information which are heavily application dependent, and briefly tell how uniformly they can be handled by ModelVisual, particularly by the Reeb graph. These applications are:

1. surface reconstruction from the contours by the homotopy model which appears in various medical and geographical visualization including computed tomography (CT) and also topographical maps;
2. volume rendering in medical visualization;
3. critical points and bifurcation in garment wrinkling modeling;
4. generating walk-through animations for human body inner trips;
5. saddle points as the critical points in forest growth visualization.

Case 1. Surface reconstruction from the contours by the homotopy model

A node \(p\) of the Reeb graph is associated with a contour (also called a contour line) \((\pi^{-1}(p))\) of the object modeled by ModelVisual. The information lost when making the equivalence class of the Reeb graph is completely recovered by this continuous geometrical information. When the object represented has a complex shape as seen commonly in natural objects, the advantage of the hierarchical modular structure of ModelVisual becomes prominent by separately storing the small key information in the top layer as the Reeb graph in the primary memory and very large geometrical information attached to it in the geometry layer in the secondary memory. In this case, the surface (the 2D manifold \(M\)) is reconstructed from a finite number of contours. This corresponds to the surface reconstruction from the contours.

Case 2. Volume rendering

A node of the Reeb graph is associated with the interior image of a contour line. The interior image can be a cross sectional images such as a CT (computed tomography) image, and thus is associated with a node of the Reeb graph. This representation finds good application in volume rendering. Medical imagery favors volume rendering techniques in reconstruct a solid object from a
given series of CT images (Herman and Liu 1979; Chen, Herman, Reynolds and Udupa 1985; Frieder, Gordon and Reynolds 1985; Levoy 1988; Drebin, Carpenter and Harahan 1988; Upson and Keeler 1988). Octrees are also used to this end (Meagher 1982; Mao, Kunii, Fujishiro and Noma 1987), and serves as another instance of the case.

**Case 3. Singularity theory, critical points and bifurcation**

A node $p$ of the Reeb graph is associated with the critical points of the contour $\pi^{-1}(p)$. First, each contour is assumed to be represented by a differentiable shape function

$$f_i : [0, 1] \to \pi^{-1}(p), \quad f_i(s) = (x, y).$$

A function

$$\Pi_x : \mathbb{R}^2 \to \mathbb{R}$$

that projects a point on the $xy$-plane to the $x$ axis is then defined as

$$\Pi_x(x, y) = x.$$

Finally, the node $p$ of the Reeb graph is associated with the critical points (local maxima and minima) and the characteristic points (the $p+$ +c singularity) of $\Pi_x(f_i)$. The Reeb graph representation is used in modeling garment wrinkling based on the singularity theory (Kunii and Gotoda 1990; Kunii and Gotoda 1991). In this model, the cylindrical coordinate system $(r, \theta, z)$ and the projection function $\Pi_r$ to the $r$ axis are used instead of the Cartesian coordinate system as in Fig. 2.

**Case 4. Walk-through animation**

A node $p$ of the Reeb graph is associated with a location inside the contour that $p$ represents. This finds application in a walk-through animation (Shinagawa, Kunii, Nomura, Okuno and Young 1990). That is, when a viewpoint is located on a point associated with $p$ and $p$ moves along the Reeb graph, we can walk through and observe the inside of the object we are visualizing. A walk-through animation is useful for the simulation of a gastroscope or a needle otoscope (Nomura 1982).

**Case 5. Saddle points as the critical points in modeling forest growth**

For botanical tree- and forest- growth visualization as shown in Aono and Kunii (1984), and Kunii and Enomoto (1991), the Reeb graph can represent the skeletons of the trees and also the pattern of forest formation processes. In this application, the critical points, particularly the saddle points (passes) of the Reeb graph are significant. In modeling tree growth, the passes correspond to the branching points, and the peaks and pits to the tips of the branches growing upward and downward, respectively. The branches growing horizontally become the singular points of the Reeb graph of the height function and should be treated separately as a special case; one immediate remedy is to rotate the axes of the height function, for example 90 degrees, to avoid this artefact of artificial singularity. To improve the modeling capability of the ModelVisual, such temporary remedy is not really desirable. Instead of the height function, we have to use a more generalized function for the Reeb graph such as the function which is defined along an arbitrary curve with a non uniform measure.

In modeling forest growth, the tree interaction through mutual shading results in either the further growth or the diminution of the trees at the various locations of a forest. Such locations are the branching points of the forest growth and become the saddle points of the Reeb graph when we use the growth function for each tree instead of the height function. The nodes that are not critical can be interpolated from the critical points. For this reason, they are derivative and can be neglected. With each saddle points, branching information (for example, the branching angle) is associated and stored in the second geometrical layer of ModelVisual.

We briefly sketched how diverse visualization cases can be uniformly modeled by ModelVisual based on the Reeb graph. The sections 6, 7 and 8 are dedicated to the further explanations of three visualization cases, 1, 3 and 5 to show the extent of the diversity we have to handle.

### 6. THE HOMOTOPY MODEL

This section is devoted to the theme of the case 1 of the previous section on the surface reconstruction from the contours which are stored in the second geometrical layer of ModelVisual as the generative information and are associated with the Reeb graph in the top topological layer of
This is a high demand area, particularly in the medical field where constructing the entire shape of a human organ from a set of cross-sections is of great significance because it is difficult to envisage the three-dimensional structure of the organ by viewing individual slices.

When a surface model is used, there are two typical ways to do this. One method approximates the contours using linear line segments and the other uses a spline function. The triangular tile techniques use the former method to generate triangular patches between contours on adjacent cross-sectional planes (see Fig. 3) (Fuchs, Kedem and Uselton 1977; Christiansen and Sederberg 1978; Boissonat 1988). Wu, Abel and Greenberg (1977) used the latter method and reconstructed surfaces with a spline approximation between the adjacent contours. The homotopy model (Shinagawa, Kunii, Nomura, Okuno and Hara 1989; Shinagawa and Kunii 1991) used a homotopy and included the two reconstruction methods described above as special cases. In this model, each contour is represented by a shape function and the surface generated between a series of contours is represented by a homotopy as the locus of the transformation (Fig. 4). This enables us to handle the problem continuously, not discretely. To be more precise, each contour on the plane \( z = z_i \) is parametrized by a variable \( s \in [0, 1] \) and represented by a function
\[
f_i : [0, 1] \rightarrow \pi^{-1}(p), \text{ as } \quad f_i(s) = (x, y).
\]
Then, the surface reconstructed between the contours on the planes \( z = z_i \) and \( z = z_{i+1} \) is represented by the homotopy
\[
F : \mathbb{R} \times [0, 1] \rightarrow \mathbb{R}^2
\]
from \( f_i \) to \( f_{i+1} \) such that
\[
F(s, 0) = f_i(s) \quad \text{and} \quad F(s, 1) = f_{i+1}(s)
\]
for all points \( s \in [0, 1] \).

The homotopy model consists of the continuous toroidal graph representation and the homotopic generation of surfaces from the representation. The continuous toroidal graph shows how each contour line is parametrized as a function and shows the correspondence between the adjacent contours.

Now we are ready to generalize the discrete toroidal graph by abstracting the features essential to reconstruct surfaces from contours. First of all, the lower and upper contours are represented by the parameters \( x \) and \( y \). The points on the contours are designated by the shape functions \( f(x) \) and \( g(y) \) whose domains are the interval \([0, 1]\). In the continuous toroidal graph, the horizontal and vertical distances between the two vertices of the continuous toroidal graph represent the differences of the parameter values between the two vertices. When a path passes through \((x,y)\) where \( f(x) \) is the coordinate value of a point \( P \) and \( g(y) \) is that of a point \( Q \), it means that \( P \) and \( Q \) is connected by a homotopy.

Generally, a surface is represented by a concatenation of monotonously increasing functions as shown in Fig. 5. The decision of this path is itself an interesting theme (Shinagawa and Kunii 1991). The toroidal graph that represents triangular patches are expressed as a concatenation of step functions. This can be considered as an approximation of the monotonously increasing functions in the discrete case. The spline approximation method is represented by the function \( y = x \) on the continuous toroidal graph. This can be considered as an approximation of the monotonously increasing functions in the piecewise continuous interpolation case. Thus, the continuous toroidal graph abstractly models the basic key feature of these typical approximation models.

Using the continuous toroidal graph representation, the homotopy model generates the surface patches between adjacent contours by connecting corresponding points on each contour by a homotopy. In other words, in this model, all the points on a contour have their corresponding points on the adjacent contours and a homotopy is used for connecting the corresponding points like fibers. Here, the correspondence is represented by the continuous toroidal graph.

Then, the homotopy model is more than the generalization of the typical surface approximation models. By carefully specifying a set of toroidal graphs, such essential surface operations as taking the first-, second- and higher- order derivatives of the surface to identify the surface properties can be defined on the surface generated by the homotopy model as needed, qualifying the homotopy model to take part in the basic model. The properties include the peaks, pits, and saddles of the surface and are serving as the nodes of the Reeb graph as explained in the section 4.1 on the
Fig. 3. Triangular tile technique

$F(z, t)$

t = 0

t = 0.3

t = 0.7

t = 1.0

$= f(x)$

$= g(x)$

Transformed

Fig. 4. Surface generation using a homotopy
Fig. 5. A path on the continuous toroidal graph

Fig. 6. Three auditory ossicles reconstructed by the homotopy corresponding to a cardinal spline surface
mountaineer's equation. The derivatives also yield the ridges and valleys which serve as the arcs of the Reeb graph.

This means the homotopy model allows ModelVisual to generate the Reeb graph as the core of the top abstract layer from the lower geometrical layer.

Fig. 6 shows the three human auditory ossicles (malleus, incus and stapes) reconstructed by using a homotopy that corresponds to the cardinal spline surface. The outline curves are approximated by the cardinal spline. Fig. 7 shows the same objects reconstructed by the Christiansen's triangulation method with the Gouraud shading. The shape looks ambiguous.

7. MODELING GARMENT WRINKLING

This section gives a closer look at the theme of the case 3 in the section 5 on singularity theory, critical points and characteristic points through the visualization modeling of a garment wrinkling phenomenon. Questions all the time asked once we started to talk about garment wrinkling were "Why wrinkling? Why not something of more scientific or industrial merit?" Indeed, to model complexity, it serves as a good example, both scientifically and industrially. For the scientific merit, of course, we could pick up the whole universe instead. The advantage of taking garment wrinkling as an example to study is in its handiness coming along with the complexity sufficient to understand its nature such as singularity.

One of the main concerns of cosmology is to model the beginning and evolution of the universe as a whole. Singularity theory is playing the central role there. Look at the initial creation of matter in the universe from the state of homogeneous energy distribution in space (see for example Wald 1984; Weinberg 1972). Since the energy is equal to the mass of matter multiplied by the light speed twice, there is a possibility to consider and model the creation of mass as the creation of a wrinkle in the energy space where homogeneous energy distribution was broken and high energy concentration is taking place at the locations where matter exists. The birth of matter is modeled by the birth of singularity. In our visualization model ModelVisual, at the highest level of abstraction, garment wrinkle creation can possibly be modeled the same simply by replacing the word 'energy' by 'cloth' and changing the scale factors. Although whether this can be actually done or not is an open problem, one of the largest challenge and temptation is testing the wrinkling modeling of the creation of the universe against the computer graphics four dimensional (4D) visualization of the astronomical observation. Challenge to open problems is the privilege of any scientists who are essentially volunteers to discover something new.

Let us now turn to the question of the industrial merit of studying garment wrinkling. In the fashion industry, garment wrinkling actually is now considered as an important key factor in garment design, especially at the highest level of design of fashion. Fashion designers try to get the most out of anything that constitutes garments. Particularly, to exploit the physical characteristics of the fabric of garments, wrinkling is occupying a major position in fashion design, for example to give a relaxed and casual atmosphere to garments when worn. The design process has a number of stages including the initial sketch by fashion designers and the extraction of patterns from the initial sketch by pattern making experts. The extracted patterns, when assembled, are expected to match the original image of the fashion designers. The traditional tools of designers have long been limited to crayons and paper. Recently several CAD systems are proposed to assist the designers with graphical editing and 3D previewing tools, but little has been done to fulfill the requirement of simulating the wrinkling behavior of garments.

In 1990, Hinds and McCartney presented a system that can display draped shape of garments. They used sinusoidal functions defined on quadrilateral patches to create folds or wrinkles. The resulting images, however, look rather stiff. Recently two major modeling techniques, usually called physically-based modeling and geometrical modeling, have been devised to model soft objects. Geometrical models (Wyvill, McPheeters and Wyvill 1986; Barr 1984) use geometrical information to represent the shape of objects. Jacobian (coordinate transformation matrix) or scalar field are examples of such geometrical information. Although geometrical models are simple and easy to program, dynamic constraints are relatively difficult to get incorporated in these models, and accordingly the resulting objects sometimes behave unrealistically. Physically-based models (Platt and Barr 1988; Terzopoulos and Fleishner 1988; Weil 1986), while computationally more complex than